Alternating Current and Resonance

When an electric current flows through a resistor in a cuicuit, some energy in the circuit is transformed into heat at a rate in proportion to the resistance of the resistor. If we sum up the energy dissipation in the resistor after any moment, the electric power P is defined as a rate of the energy dissipation $\mathcal{E}_R$ and elapsed time t as $P = \frac{d\mathcal{E}_R}{dt}$. This electric power P is also given by a formula $P = V \cdot i$, where V is a voltage drop across the resistor and i is the current in the circuit. Additionally, Ohm's law gives the proportionarity between the voltage drop V and the current i, i.e., $V = R \cdot i$, where R is the resistance.

The amount of voltage V, current I, resistance R, power P and energy dissipation $\mathcal{E}_R$ should be given in units of Volt (symbol: V), Ampere (A), Ohm (Ω), Watt (W) and Joule (J), respectively. We should check unit relations in the above fomulae so as to

$$P = \frac{d\mathcal{E}_R}{dt} \qquad [\mathrm{W}] = [\mathrm{J/s}],$$
$$P = V \cdot i \qquad [\mathrm{W}] = [\mathrm{V} \cdot \mathrm{A}], \quad \text{and}$$
$$V = R \cdot i \qquad [\mathrm{V}] = [\Omega \cdot \mathrm{A}].$$

A coil is a device with a cylindrical space filled with magnetic fields that are generated by an electric current which flows along a single long wire that is closely wound in many turns in one direction. A capacitor is a device with a thin space filled with electric fields that are generated by a set of plus-and-minus charges which are stored on a set of top-and-bottom planes of the space. At an ideal coil or capacitor, none of energy is dissipated from the system of the circuit. The energy is stored in the inner space of them and can be supplied back into the circuit.

The coil and capacitor in a circuit can exchange their energies and then an alternating current with some frequency is naturally generated. This natural frequency must be given by the circuit constants of the coil and the capacitor. Moreover, when an oscillating electromotive force is applied on the circuit, the alternating current with current source frequency can happen to be larger and larger because the energy can be accumulated in the coil and the capacitor. This phenomena is called as a resonance.

In an analog radio, for example, turning the tuning knob changes some circuit constant and chages the natural frequency. Eventually, one can set the natural frequency to that from the broadcast. This enables a faint energy of applied electromotive force on the antenna from a broadcast accumlated into the radio circuit, so as to make a detectable signal.

Experiment 1: Coil and Capacitor

Purpose 1: Using an oscilloscope, to observe the voltage drop of a coil and a capacitor along which an alternating current flows and to identify the phase shifts.

Theory 1-1: Coil and its self-inductance

The excited magnetic energy $\mathcal{E}_L$[J] in the coil, that is generated by the current i[A], is given by $\mathcal{E}_L = \frac{1}{2} L\, i^2$, where L is a self-inductance of the coil. The unit of L is Henry (Symbol: H) which is defined as $[\text{H}] = [\text{J/A}^2] = [\text{s} \cdot \text{V/A}] = [\text{s} \cdot \Omega]$.

From the formula $\mathcal{E}_L = \frac{1}{2} L\, i^2$, the energy transfer ratio $P = \frac{d\mathcal{E}_L}{dt} = L\frac{di}{dt} \cdot i$ should give the power, $P = V_L \cdot i$. These equations indicate that the voltage drop V_L should be given by $V_L = L\frac{di}{dt}$.

Then, let us consider the case of the alternating current $i(t) = i_0 \sin(\omega t)$ which flows through the coil. Here, i_0 is the amplitude of the current and ω is the angular frequency. The voltage drop is given by

$$V_L = L\frac{di}{dt} = \omega L\, i_0 \cos(\omega t) = \omega L\, i_0 \sin(\omega t + \tfrac{\pi}{2}).$$

This voltage drop is said to advance in phase by an angle of $\pi/2 = 360^\circ/4$ in comparison to the current $i_0 \sin(\omega t)$, because the current at $t = 0$ and the voltage drop at $t = -\pi/\omega\,(< 0)$ are in the same phase.

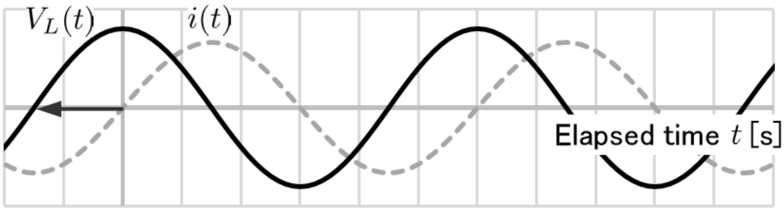

Fig.1. The curve for the voltage drop at a coil which shifts toward the past time.

Theory 1-2: Capacitor and its capacitance

The excited electrostatic energy $\mathcal{E}_C$[J] in the capacitor, that is generated by the applied voltage V[V], is given by $\mathcal{E}_C = \frac{1}{2} C\, V^2$, where C is a capacitance of the capacitor. The unit of C is Farad (Symbol: F) which is defined as $[\text{F}] = [\text{J/V}^2] = [\text{s} \cdot \text{A/V}] = [\text{s}/\Omega]$.

From the formula $\mathcal{E}_C = \frac{1}{2} C\, V^2$, the energy transfer ratio $P = \frac{d\mathcal{E}_C}{dt} = V \cdot C\frac{dV}{dt}$ should give the power, $P = V_C \cdot i$. Here, the applied voltage on the capacitance is a part of applied voltage from the source (battery) and is measured as the voltage drop. Then, these equations indicate that the current i should be given by $i = C\frac{dV}{dt}$.

Let us consider the case of the alternating current $i(t) = i_0 \sin(\omega t)$ which flows through the capacitor. The voltage drop V_C should satisfy the relation $C\frac{dV}{dt} = i_0 \sin(\omega t)$ so that

$$V_C = \frac{1}{C}\int i_0 \sin(\omega t)\, dt = -\frac{1}{\omega C}\, i_0 \cos(\omega t) = \frac{1}{\omega C}\, i_0 \sin(\omega t - \tfrac{\pi}{2}).$$

This voltage drop is said to be delayed in phase by an angle of $\pi/2 = 360^\circ/4$ in comparison to the current, because the current at $t = 0$ and the voltage drop at $t = \pi/\omega\,(> 0)$ are in the same phase.

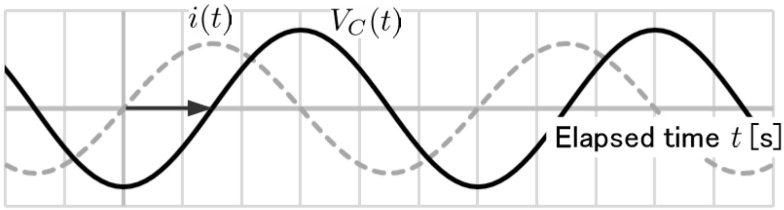

Fig.2. The curve for the voltage drop at a capacitor which shifts toward the future.

Procedures 1:

1. Make an LR circuit shown in Fig.3(a) using an LCR block and a bypass lead by which removes any current of the capacitor so that it is eliminated from the circuit.

2. Connect two probes of an oscilloscope: one with red ring, measuring V_L as shown in left hand side in Fig 3(a), should connect to the upper terminal: channel 1 (CH1), and the other with blue ring, measuring V_R as shown in right hand side in the same figure, should connect to the lower terminal: channel 2 (CH2). One should remark that both clips of probes are connected to the same place in the circuit, because they are connected in the scilloscope and referred to as 0 V.

3. Make the graph of CH2 inversed. In case of the oscilloscope HDS1022M, push the MENU button and select 'CH1 setup' pushing $\triangle$ and $\triangledown$ buttons, then F4 button becomes a toggle button of INVERSE ON and OFF.

4. Set the output of a signal generator to a 16 kHz sinusoidal wave and connect as the alternating power source. Push AUTOSET button on the oscilloscope. Record the graph shapes on the display of the oscilloscope and identify their characteristics.

5. Read the amplitudes of CH1 and CH2, that are $\omega L i_0$ and $R i_0$, respectively. Measure the R of the resistor using a digital multimeter (DMM) and estimate the self-inductance L in the unit $[\mathrm{H}] = [\mathrm{s} \cdot \Omega]$ at $\omega = 2\pi \times 16\,\mathrm{kHz}$.

6. Make an CR circuit as shown in Fig.3(b). Connect two probes of an oscilloscope: one with red ring, measuring V_C, should connect to CH1, and the other with blue ring, measuring V_R, should connect to CH2 and. Confirm that only the graph of CH2 is INVERSE ON. Push AUTOSET button on the oscilloscope. Record the graph shapes on the display of the oscilloscope and identify their characteristics.

7. Read the amplitudes of CH1 and CH2, that are $i_0/\omega C$ and $R i_0$, respectively. Estimate the capacitance C in the unit $[\mathrm{F}] = [\mathrm{s}/\Omega]$.

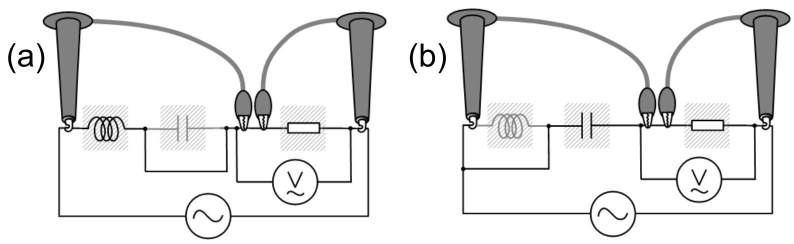

Fig.3. Usage of LCR block for LR circuit (a) or CR circuit (b).

Experiment 2: Resonance of LCR circuits

Purpose 2: To draw an resonance curve that indicates that the resonance frequency coincides with a characteristic frequency which is given by circuit constants of the coil and capacitor in the circuit.

Theory 2-1: LC series resonance

Let us consider that an alternating electromotive force with angular frequency ω ($= 2\pi f$, where f [Hs] is the frequency) is applied to an LC series circuit (Fig.4). An output current $i(t) = i_0 \sin(\omega t)$ from the alternating-current source flows through all of the the coil with self-inductance L [Hz] ($=$ [s $\cdot\, \Omega$]), the capacitor with capacitance C [F] ($=$ [s/Ω]) and the resistor with resistance $R\,[\Omega]$.

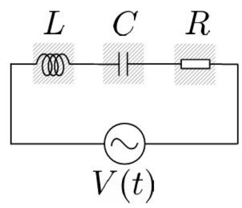

Fig.4. An LC series circuit with a resistor R.

The output voltage of the current source should be written as $V(t) = V_0 \sin(\omega t + \phi)$, where both the voltage amplitude V_0 and the phase difference ϕ depend on the frequency. One can derive the analytical formulae of them as

$$V_0 = \left(\omega L - \frac{1}{\omega C}\right)\frac{i_0}{\sin\phi} \qquad \text{and} \quad \tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

or

$$i_0 = \frac{V_0}{R}\frac{1}{\sqrt{1 + Q^2(x - \frac{1}{x})^2}} = \frac{V_0}{R}\cos\phi \qquad \text{and} \quad \tan\phi = Q\left(x - \frac{1}{x}\right)$$

where $x = \omega\sqrt{LC}$ and $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$. The frequency dependences of $\tan\phi,\ \phi,\ \cos\phi$ are shown in Fig.5. The curve of $\cos\phi$, that indicates current amplitude i_0, is called as a resonance curve for LC series circuit.

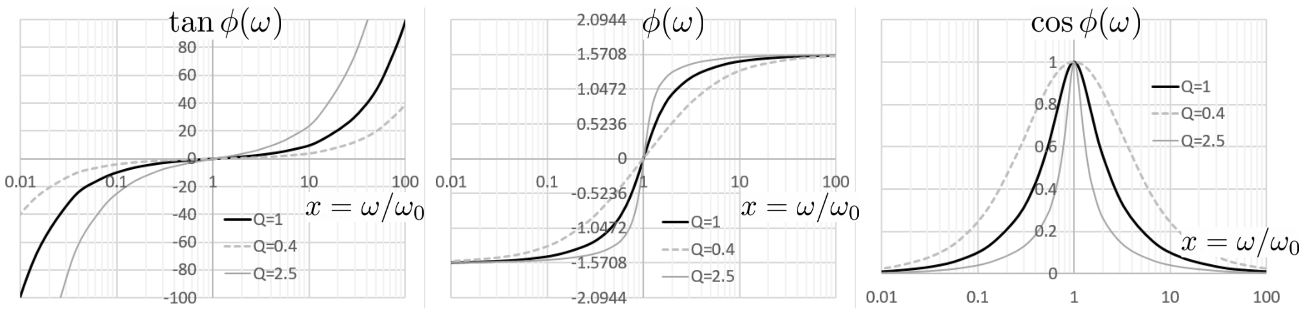

Fig.5. The frequency dependence of $\tan\phi,\ \phi,\ \cos\phi$ of LC series circuit. The horizontal axis of those graphs are $x = \omega\sqrt{LC}$ in a log scale.

The characteristic frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \text{or} \qquad f_{\rm res} = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

is called a resonance frequency. The resonance curve of LC series circuit shows

- $i(t) \sim 0$ at $f \ll f_{\rm res}$ or $f \gg f_{\rm res}$ (off-resonance),
- $i(t) = V(t)/R$ at $f = f_{\rm res}$ (resonance).

Theory 2-2: LC parallel resonance

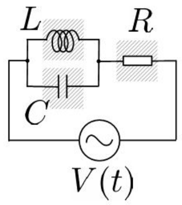

Fig.6. An LC parallel circuit with a resistor R.

Let us consider that an alternating electromotive force with angular frequency ω ($= 2\pi f$, where f [Hs] is the frequency) is applied to an LC parallel circuit (Fig.6). An output current $i(t) = i_0 \sin(\omega t)$ from the alternating-current source flows through the resistor R but it separates into i_L and i_C along the the coil and the capacitor, respectively, where $i = i_L + i_C$.

The output voltage of the current source should be written as $V(t) = V_0 \sin(\omega t + \phi)$, where both the voltage amplitude V_0 and the phase difference ϕ depend on the frequency. One can derive the analytical formulae of them as

$$V_0 = \frac{1}{\frac{1}{\omega L} - \omega C} \frac{i_0}{\sin\phi} \quad \text{and} \quad \tan\phi = \frac{1}{R} \frac{1}{\frac{1}{\omega L} - \omega C}$$

or

$$i_0 = \frac{V_0}{R} \frac{1}{\sqrt{1 + Q^2 \frac{1}{(\frac{1}{x} - x)^2}}} = \frac{V_0}{R} \cos\phi \quad \text{and} \quad \tan\phi = Q \frac{1}{\frac{1}{x} - x}$$

where $x = \omega\sqrt{LC}$ and $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$. The frequency dependences of $\tan\phi,\ \phi,\ \cos\phi$ are shown in Fig.7. The curve of $\cos\phi$, that indicates current amplitude i_0, is called as a resonance curve for LC parallel circuit.

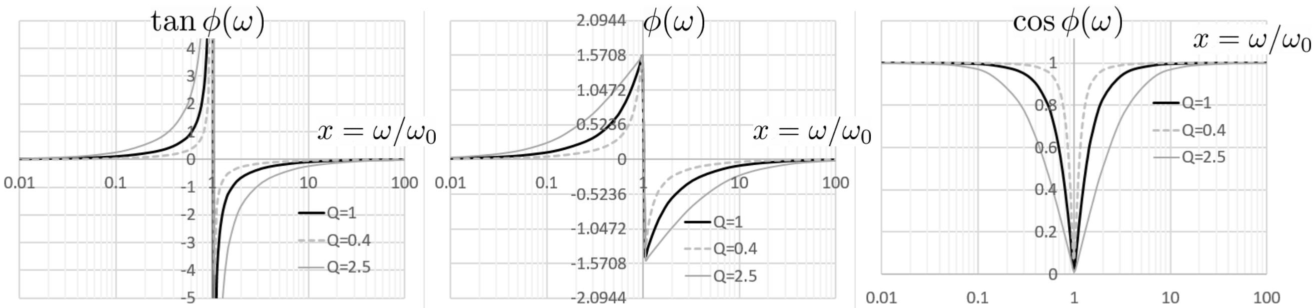

Fig.7. The frequency dependence of $\tan\phi,\ \phi,\ \cos\phi$ for LC parallel circuit. The horizontal axis in those graphs is $x = \omega\sqrt{LC}$ in a log scale.

The characteristic frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_{\text{res}} = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

is also called a resonance frequency. The resonance curve of LC parallel circuit shows

- $i(t) \sim V(t)/R$ at $f \ll f_{\text{res}}$ or $f \gg f_{\text{res}}$ (off-resonance),
- $i(t) = 0$ at $f = f_{\text{res}}$ (resonance).

Procedures 2:

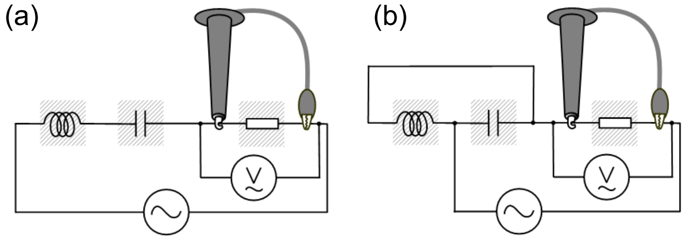

Fig.8. An LC series circuit (a) and pararell circuit (b) by use of the LCR block.

1. Using the LCR block, make arrangement of the LC series or parallel circuit shown in Fig.8 (a) or (b). Connect probes of the oscilloscope in order to observe the waveform of the voltage drop at resistance V_R, and connect probes of the DMM in order to read the effective value of V_R, which is $2\sqrt{2}$ times smaller than an peak-to-peak amplitude of V_R.

2. Connect the alternating-current source at the appropriate points in the circuit. Make sure that an additional lead is needed to set the LC parallel circuit.

3. Set the current source frequency at the values in the below table which is appropriate if the resonance frequency is close to 16 kHz. Read the effective value of V_R by use of the DMM or the peak-to-peak amplitude of it by the oscilloscope. Take care that the upper frequency limit of measureable AC current frequency is about 20 kHz for the DMM PC-510 and 20 MHz for the oscilloscope HDS1022M.

Table.1 A list of frequencies in Hz for the resonance curve experiments if $f_{\mathrm{res}} \sim 16\,\mathrm{kHz}$.

50	100	200	500	1k	2k	4k	6k	8k	9k
10k	11k	12k	13k	14k	15k	16k	17k	18k	19k
20k	23k	30k	50k	100k	200k	500k	1M		

4. Plot the resonance curves on semi-log graph sheets using the results. Determine the resonance frequency by folding the sheet in half and overlaying the right and left parts of resonance curves.

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