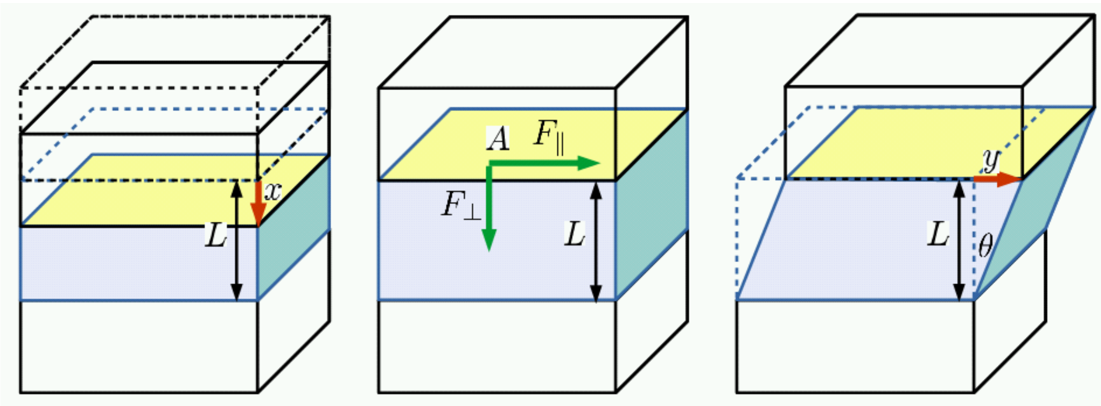
Elasticity

Forces on the planar surfaces at which two solid bodies are in contact with each other can be decomposed into perpendicular and parallel components. The former is a **normal force** and the latter is a **frictional force**. Let us think the case the other thin **elastic body** is sandwithed between these two bodies. Throughout the sheet-like body, the uniform forces of pressing (perpendicular) and shearing (parallel) are transmitted, and as a result, the deformation is also uniform. The amount of shrinking is represented as **normal strain** $\varepsilon_\perp$, which is defined to be the perpendicular displacement of the surface divided by the tickness of the elastic sheet. The amount of shearing is shown as **shear strain** $\varepsilon_\parallel$, that is the parallel displacement divided by the thickness. These strains on the elastic bosy are shwon in Fig.A-1. These strains appear due to the **normal stress** $\sigma_\perp$ or **shear stress** $\sigma_\parallel$, which are defined as the normal or frictional forces divided by the area of the surface on which these forces are applied.

Although there are some exceptions, many kinds of material shows the linear relations that the normal strain $\varepsilon_\perp$ arises due to the normal stress $\sigma_\perp$ with a proportionality $\sigma_\perp = E\varepsilon_\perp$, and the shear strain $\varepsilon_\parallel$ arises by the shear stress $\sigma_\parallel$ under the relation $\sigma_\parallel = G\varepsilon_\parallel$. Here, those strains should be sufficiently small ($\varepsilon_\perp \ll 1$ and $\varepsilon_\parallel \ll 1$). The coefficience E is called **Young's modulus**, and G the **modulus of rigidity**. The elastic quantities with their uints are summerized in Table A-1.

The stress-strain propotionality is known as **Hooke's law**. Usually this law is applied for a spring and written as $F = kx$, where x is increase of the length of the spring by the tensile force F, and the spring constant k indicates the hardness of the spring.



Normal strain

Normal and frictional forces applied on the sandwitched elastic body with thickness L

Shear strain

Fig. A-1 Normal and shear displacements and strains by forces

Here we did not take into account any anisotropy. In addition, some textbooks use a different definition of the shear strain as $\gamma = 2\varepsilon_{\|}$. Typical values of the elastic moduli for some materials in round numbers are shown in Table A-2. It is known that components addition in steel or glass can change those elastic moduli considerably.

Because those moduli are quite large in comparison with, for example, the standard atmospheric pressure $P_{\mathrm{std}} = 101\,325 \sim 10^5\,\mathrm{N/m^2}$, the strain $\varepsilon_\perp$ or $\varepsilon_{\|}$ should be always quite small. However, if the material length (thickness) L is sufficiently large, the displacement $x = L\varepsilon_\perp$ or $y = L\varepsilon_{\|}$ is able to be observed.

Table. A-1 Definitions of elastic quantities

force [N]	stress [N/m²]	displacement [m]	strain [dimensionless]	elastic modulus [N/m²]
normal force $F_\perp$	normal stress $\sigma_\perp = F_\perp/A$	normal displacement x	normal strain $\varepsilon_\perp = x/L$	Young's modulus $E = \sigma_\perp/\varepsilon_\perp$
frictional force $F_{\|}$	shear stress $\sigma_{\|} = F_{\|}/A$	parallel displacement y	shear strain $\varepsilon_{\|} = y/L (= \theta)$	modulus of rigidity $G = \sigma_{\|}/\varepsilon_{\|}$

Table. A-2 Typical values of the elastic moduli in round numbers

$[\times 10^9\,\mathrm{N/m^2}]$	diamond	steel	copper	glass	polyethylene	rubber
Young's modulus E	1200	200	120	70	0.5	0.1
modulus of rigidity G	480	80	50	30	0.1	0.001

Experiment 1: Young's Modulus of a Wire (Searle's Apparatus)

Purpose: To evaluate Young's modulus of a metal wire by measuring small stretch by use of Searles framework.

Theory 1-1: Searle's framework with a sample and auxiliary wires

Figure 1-1 shows Searle's framework inteded to measure stretch of a sample wire AA' which is installed in parallel to an auxiliry wire BB'. When a load on H is incleased, a level L is inclined, which should be recovered to keep horizontal by screw D. The change of the position of D is read using the scale S, which corresopnds to the stretch of the sample wire.

Procedures:

1. Put one weight on both of the hooks H and K. and measure the length L of AA' by a scale to a precision of 1 mm. Measure the diameter d by averaging two-direction reads (a and b in Fig. 1-2) and their repetitions at several places along the wire by use of a micrometer to a precision of $1\,\mu$m.

2. Adjust the level L to be horizontal by rotating the screw D and read the scale S to a precision of $1\,\mu$m. This read is named as z_0.

3. Add one weight and adjust the level to be horizontal again, and read the scale as z_1. Repeat measurements $z_2, z_3, \cdots, z_5$ for up to five additional weights. Wait several minutes and measure the scale again without change of the weights. This read, z_5', can not the same with z_5. Repeat measurements $z_4', z_3', \cdots, z_0'$ with decreasing number of weights.

4. Make a graph to plot the average $\bar{z}_i = (z_i + z_i')$ versus the load W_i/g. Fit the plots to a line and obtain the inclination. It shows the averaged value of the ratio of load W (or their mass W/g using $g = 9.80\,\mathrm{m/s^2}$) and the stratch ΔL, i.e., $\langle (W/g)/\Delta L \rangle$.

5. Confirm that there remains the first weight which is hanged when you measure the length L and the diameter d. Measure them again and calculate their averages between before and after experimets. Calculate the Young's modulus

$$E = \frac{\sigma_\perp}{\varepsilon_\perp} = \frac{W/(\pi d^2/4)}{\Delta L/L} = \frac{4Lg}{\pi d^2}\left\langle \frac{W/g}{\Delta L} \right\rangle.$$

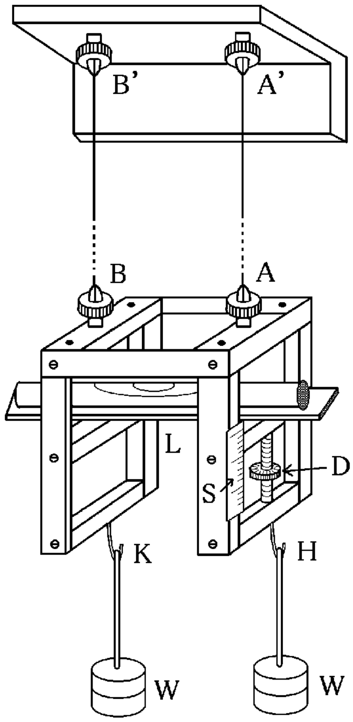


Fig.1−1 Searle's framework

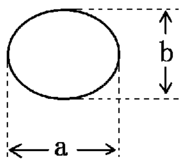


Fig.1−2 Diameter measurements

Sample Data:

i	Mass W_i/g [kg]	Scale reading z_i (increasing)	Scale reading z_i' (decreasing)	Average $\bar{z}_i = \dfrac{z_i + z_i'}{2}$
0	0.998	19.515	19.522	19.519
1	1.996	19.272	19.445	19.279
2	2.994	19.001	19.006	19.004
3	3.992	18.764	18.758	18.761
4	4.989	18.535	18.536	18.536
5	5.988	18.276	18.312	18.294

Experiment 2: Young's Modulus of a Bar (Ewing's Apparatus)

Purpose: To evaluate Young's modulus of a metal bar by measuring a very small deflection by use of an optical lever technique.

Theory 2-1: Deformation of a flat metal bar

A flat square metal bar is placed on two sharp edges, as shown in Fig.2-1(a). A load W is applied at the right middle of the edges. Both of the normal forces on the edges are half in size, i.e., $W/2$. Figure 2-1(b) shows that the bar deflects due to these forces which are applied vertically, and then almost all the parts in the bar do more or less move downwards. Nevertheless, if we subdivide the bar into small cells with approximately uniform deformation (those around a point P in Fig. 2-1(b) are shown in Fig. 2-2(c) and (d)), it is easy to understand that those in the upper and lower half of the bar are horizontally compressed and stretched, respectively. This means that such deflection is a result of integration of the inner longitudional elasitic strain, and proportionality of the load W and deflection e, i.e., the vertival displacement of the center of the bar, will give Young's elastic modulus E of the material. For the bar with the shape of thickness a and width b that is placed on the edges separated by a distance L, a forn

$$W = \left(\frac{4a^3b}{L^3}E\right)e \tag{1.1}$$

is given by a calculus analysis which is shown in Appendix 2-A.

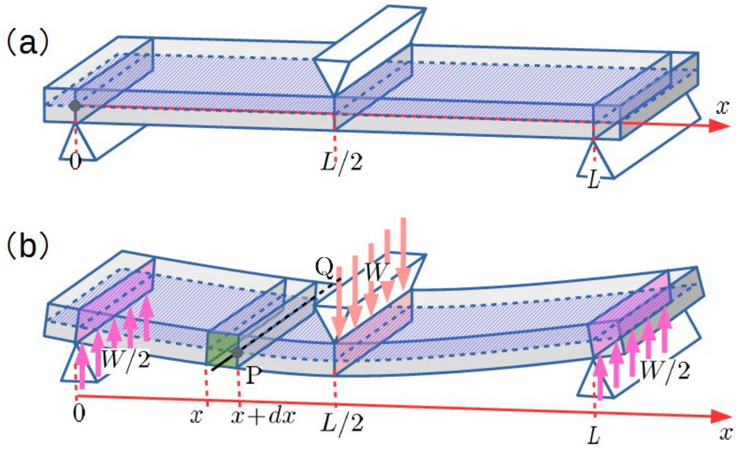


Fig.2−1 The deformation of a flat metal bar.

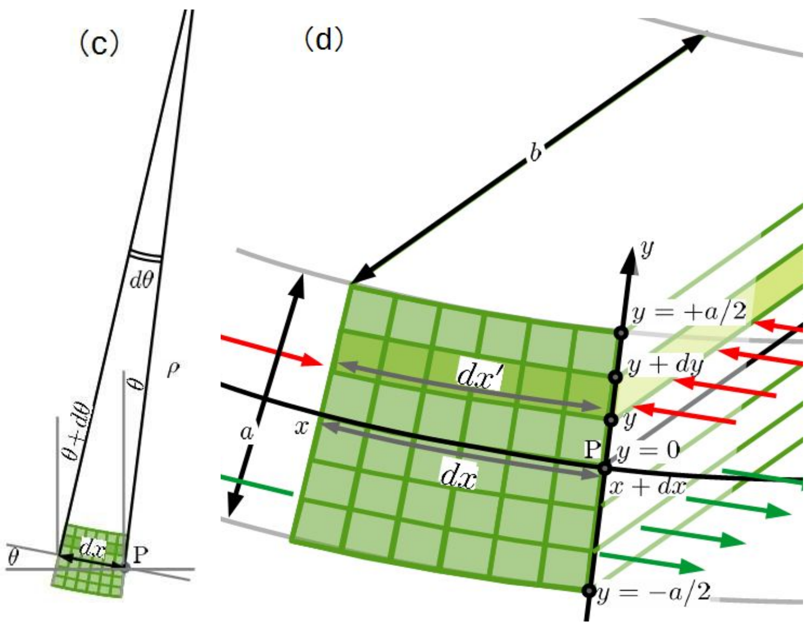


Fig.2−2 Enlarged viewa of the bar (x to x+dx).

Theory 2-2: Optical lever

The quite small distance e must be measured without any touch because of the stability of the system. A scottish physist James Alfred Ewing had created a clever way of the measurement, so called 'an optical lever'. The distance e is transformed into an angle α of an inclination of a millor which is placed on the specimen bar and a undeformed bar placed in parallel to the specimen. One can read a vertical scale using this millor from a telescope installed on the scale. A large ratio of the length between the millor and the scale and that between two bars makes easy to find the amount of the deflection, e.

Appendix 2-A: Elastic relation of deflection and load of metal bar

The normal stress $\sigma_\perp$ on a sufficiently small element of the deflected bar can be considered to be uniform, though it is non-uniform for the whole bar. The normal strain $\varepsilon_\perp$ that can be expected from the shape of the bar can give the stress by a relation $\sigma_\perp = E\varepsilon_\perp$ where E is Young's modulus. In order to describe the distribution of such non-homogeneous quatities, we put a coordinate x along the length of the bar, as shown Fig. 2-1(a). A small segment of the bar from x to $x + dx$ is uniformly curved. In addition, a coordinate y is defined to the direction to the center of curvature, then the top and back surface of the bar is located at $y = +a/2$ and $-a/2$, respectively. The radius of the curvature for $y = 0$ is ρ and that for y is $\rho - y$. The angle of view from the center of curvature is $|d\theta|$. A horizontal slice of the segment with length $dx = \rho|d\theta|$, located at y with width dy in the upper half ($y > 0$), is shrunk to a length $dx' = (\rho - y)|d\theta|$ along the x axis. Then the strain $\varepsilon_\perp$ is given as

$$\varepsilon_\perp = \frac{dx' - dx}{dx} = \frac{(\rho - y)|d\theta| - \rho|d\theta|}{\rho|d\theta|} = -\frac{y}{\rho}\,.$$

This form can be applied to the unchanged slice ($y = 0$, $\varepsilon_\perp = 0$) or stretched slice in the lower part ($y < 0$, $\varepsilon_\perp > 0$). Here, the radius ρ changes with the x coordinate. We need to resolve the x dependence of ρ from the viewpoint of mechanics.

A slice (y to $y + dy$) of the segment (x to $x + dx$) has a vertical surface with area bdy located at $x + dx$. There normal stress $\sigma_\perp = E\varepsilon_\perp = -Ey/\rho$ is thought to be applied. Then, a total force $F_\perp$ due to the normal stress $\sigma_\perp$ on the vertical surface with the area ab becomes zero :

$$F_\perp = \int \sigma_\perp dS = \int_{y=-a/2}^{y=+a/2} \left(\frac{Ey}{\rho}\right)(bdy) = \frac{Eb}{\rho}\int_{y=-a/2}^{y=+a/2} ydy = 0$$

but a total torque M due to the normal stress around a horizontal axis at $y = 0$ (the point P or the axis PQ) is not zero:

$$M = \int y(\sigma_\perp dS) = \int_{y=-a/2}^{y=+a/2} y\left(\frac{Ey}{\rho}bdy\right) = \frac{bE}{\rho}\int_{y=-a/2}^{y=+a/2} y^2 dy = \frac{a^3}{12}\frac{bE}{\rho}\,.$$

On the left part of the bar from the point P, the torque M should be calcelled by the other torque M' due to the applied upward normal force $W/2$ at $x = 0$, as shown in Fig.2-3. The distance of the force from the point P is $x + dx \sim x$. Then,

$$M' = -(W/2)x.$$

The resultant torque around P is $M + M' = 0$. Then the radius ρ of the curvature is found to be inversely proportional to x, i.e.,

$$\frac{1}{\rho} = \frac{6W}{a^3 bE}x.$$

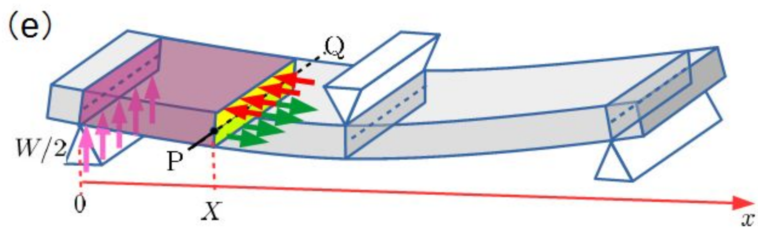


Fig.2-3 Torque balance of a part (0 to x+dx)

Lastly, the vertival displacement of the center of the bar e is calculated as the summension of the vertical distance of the segment with the length dx at x, which has a small slope of $\theta(x) \ll 1$ from the horizontal line, so that

$$e = \int_{x=0}^{x=L/2} dx \sin\theta \sim \int_{x=0}^{x=L/2} \theta(x) dx$$

The difference of angles $\theta + d\theta$ and θ for the neighbouring segments at x and $x + dx$ equals to $d\theta = -dx/\rho$, which can be integrated as

$$\theta = \int_{(\theta=0)}^{(\theta)} d\theta = \int_{x=L/2}^{x} \left(-\frac{dx}{\rho}\right) = \frac{6W}{a^3bE} \int_{x}^{x=L/2} x\,dx = \frac{3W}{a^3bE}\left(\frac{L^2}{4} - x^2\right).$$

Then,

$$e = \frac{3W}{a^3bE}\left[\frac{L^2}{4}x - \frac{x^3}{3}\right]_0^{L/2} = \frac{3W}{a^3bE}\frac{L^3}{12} = \frac{L^3}{4a^3bE}W$$

which is the same as the elastic relation of eq.(1-1).

Apparatus and Procedures:

1. Put two metal bars (sample AB and auxiliary CD) in parallel on the knife edges AC and BD, as shown in Fig.13. Put a hanger E with a millor G and three legs P, Q, R on these bars. Mount some piece of weights on the hanger. Adjust the direction of the millor G which should look upward slightly.

2. Adjust the telescope to the same level of the millor. Place the scale with the telescope apart from the millor G with a large distance D (around 3 m). Move its position where the scale is found at the center of the visual field through the telescope as a reflected image by the millor. Adjust the inclination of the millor so that the scale reading without weight ($z_{\min}$) and that with full weight ($z_{\max}$) are all available.Figure 11 shows that this apparatus works as an optical lever which enebles to measure the deflection e by difference of the scale reading Δz.

$$e = h\alpha = \frac{h}{2D}\Delta z.$$

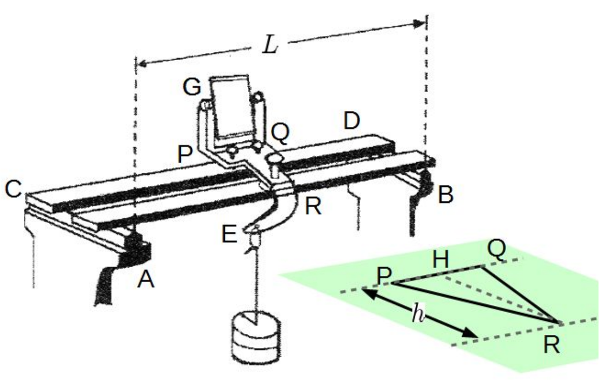


Fig.2-4 Bars and edges.

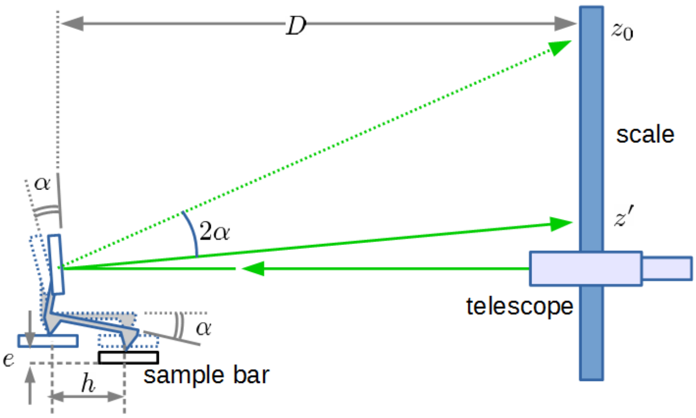


Fig.2-5 Optical lever.

3. Start the measurement. Let z_0 be the scale read under the load of two weights. Readthe scale under the load with increased number of weights one by one, and get $z_1, z_2, \cdots, z_5$. Again, Read the scale under the load with decreasing number of weights one by one, and get z'_5, z'_4, $\cdots$, z'_0. Find the mass of individual weights and calculate the loads W_i and averages $\bar{z} = (z + z')/2$ for each measurement. Make a graph to plot the displacement $\Delta z = \bar{z} - \bar{z}_0$ versus mass of load W/g. Find the inclination $< (W/g)/\Delta z >$.

4. Measure a set of three lengths between P and Q, Q and R, R and P by use of a varnier caliper for three times and calculate their averages. The triangle PQR should be identified as an isoscales triangle so that the length h can be obtaind by

$$h = \frac{1}{2}\sqrt{(\mathrm{PR} + \mathrm{QR})^2 - \mathrm{PQ}^2}.$$

5. Measure the thickness a and width b of the sample bar by use of a micrometer. Measure the distance D and the length between the edges L.

6. Calculate the Young's modulus by

$$E = \frac{L^3 D g}{2a^3 b h}\left\langle \frac{W/g}{\Delta z} \right\rangle.$$

Sample Data:

i	Mass W_i/g [g]	Scale reading z_i (increasing)	Scale reading z'_i (decreasing)	Average $\bar{z}_i = \dfrac{z_i + z'_i}{2}$
0	0	21.3	21.5	21.4
1	201.84	87.1	87.4	87.3
2	403.43	153.5	153.7	153.6
3	604.13	218.9	218.8	218.9
4	804.70	248.8	384.4	284.6
5	1003.83	353.8	353.7	353.8

$g = 9.80665\,\mathrm{m/s^2}$,

$D = 3.093\,\mathrm{m}$,
$a = 4.017 \times 10^{-3}\,\mathrm{m}$,
$b = 16.082 \times 10^{-3}\,\mathrm{m}$,
$L = 400.8 \times 10^{-3}\,\mathrm{m}$,
$h = 29.345 \times 10^{-3}\,\mathrm{m}$,
$E = 9.698 \times 10^{10}\,\mathrm{N/m^2}$

Experiment 3: Modulus of Rigidity of a Wire in a Torsion Pendulumn

Purpose: To evaluate the modulus of rigidity of brass by measuring the a couple of periods of torsion pendula.

Theory 3-1: Torsional Deformation and Force of Restitution of a Wire

A wire with the modulus of rigidity G, the radius R and the length L is clamped at the top and hanged vertically. A circumferential external force F_{ex} which is applied at a point P on the edge of the bottom surface made it rotates by an angle α[rad], as shown in Fig. 3-1. A uniform shear deformation occurs within a cylinder of length L, the radius r and the thickness dr. Because it's shear displacement is $r\alpha$, the shear strain is

$$\varepsilon_{\|} = r\alpha/L\,(=\theta). \qquad (3.01)$$

This should give the shear stress

$$\sigma_{\|} = G\varepsilon_{\|} = r\alpha G/L. \quad (3.02)$$

The area of the bottom surface of the cylinder is

$$dA = 2\pi r dr, \qquad (3.03)$$

so the shear force on the cylinder can be written from the formula $df = -\sigma_{\|} dA$ as

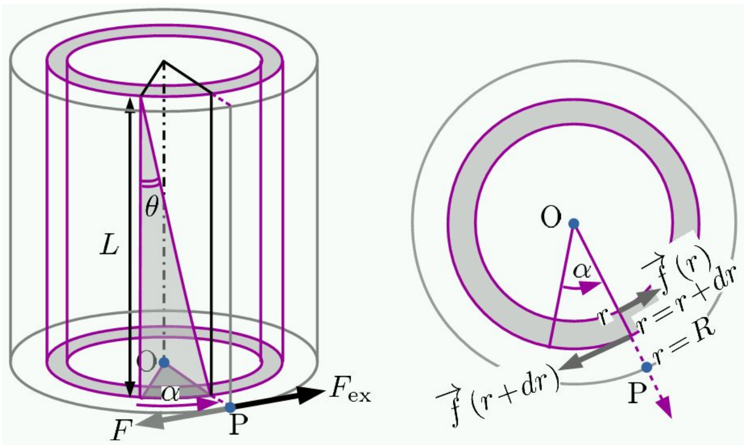


Fig. 3-1 Torsional deformation of a wire

$$df = -r\alpha G/L \times 2\pi r dr = -\left(2\pi\alpha G/L\right) r^2 dr. \qquad (3.04)$$

We put a negative sign because here $f(r)$ indicates the sum of the force of restitution, that is always negative when the positive direction was the same with the angle α, on the bottom surface of a column with a radius r (the central part of the wire). Therefore, $f(R) = -F$, which is equal in magnitude and has the opposite direction of the external force F_{ex} on P.

The torque $d\tau$ on the bottom area dA of the cylinder (eq.3.03) around the central axis is

$$d\tau = r df = -\left(2\pi\alpha G/L\right) r^3 dr. \qquad (3.05)$$

Then, the net torque is given as

$$\tau^{\mathrm{net}} = \int_{r=0}^{r=R} d\tau \quad = -\kappa\alpha, \text{ where } \kappa = \frac{\pi G R^4}{2L} \qquad (3.06)$$

This equation means the Hooke's law between the rotational force of restitution (torque τ^{net}) and displacement (angle α).

Theory 3-2: Dynamics of Rotational Oscillation

In order to put a ring-shaped weight horizontally (Fig.3-2(b)) or vertically (Fig.3-2(c)), a fitting (Fig.3-2(a)) is connected on the bottom of the wire. A system consisting of the weight and the fitting rotates around the vertical axis up to a limit angle because of the torsional force of restitution by the wire. Here we define an inertial moment of the fitting is I^{fitting}, The formulas of inertial moments of the doughnut-shaped weight shown in Fig.3-2(d) with the inner radius a, the outer radius b, the thickness c and mass M are known as

$$I^{\text{horiz}} = I_{zz} = \frac{M}{2}\left(a^2 + b^2\right) \quad \text{around the vertical axis in Fig.3-2(d), and} \qquad (3.07)$$

$$I^{\text{vert}} = I_{xx} = \frac{M}{12}\left(3a^2 + 3b^2 + c^2\right) \quad \text{around a horizontal axis there.} \qquad (3.08)$$

Later we use a formula of the difference

$$I^{\text{horiz}} - I^{\text{vert}} = \frac{M}{12}\left(3a^2 + 3b^2 - c^2\right). \qquad (3.09)$$

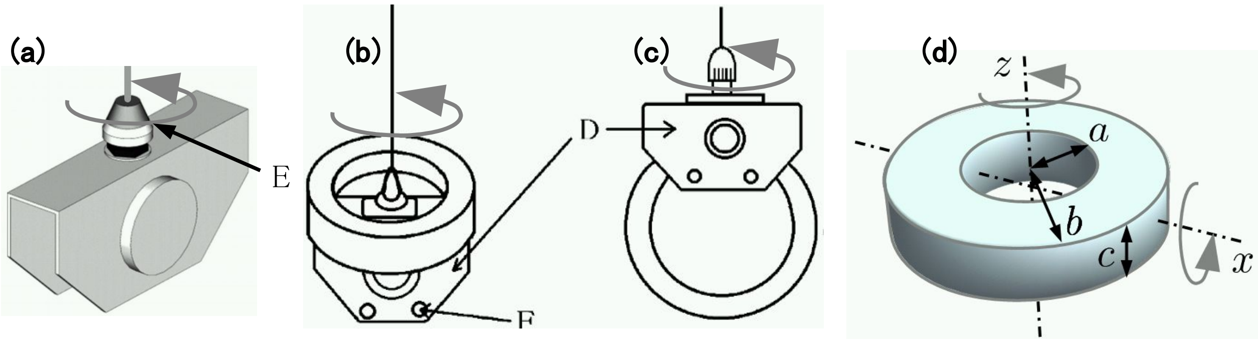


Fig. 3-2 (a) A fitting for the bottom of the wire. (b) A weight placed horizontally on the fitting and (c) the same weight hanged vertically. (d) The dimensions of the ring-shaped weight.

In the rotational motion, the angular acceleration $d^2\alpha/dt^2$ of the system is in proportion to the net torque τ^{net} by the wire, that is given by eq.3.06. The equation of motion is then

$$I\, d^2\alpha/dt^2 = \tau^{\text{net}} = -\kappa\alpha \qquad (3.10)$$

where $I = I^{\text{horiz}} + I^{\text{fitting}}$ for Fig.3-2(b) and $I = I^{\text{vert}} + I^{\text{fitting}}$ for Fig.3-2(c). The solution of eq.3.10 is a simple harmonic oscillation with a period T written as

$$\alpha = C \sin(2\pi t/T + \phi)\,, \qquad (3.11)$$

where C and ϕ are constants which are determined by the initial condition of motion. The period T need to satisfy a relation $(2\pi/T)^2 = \kappa/I$, therefore,

$$\left(T^{\text{horiz}}\right)^2 = \frac{4\pi^2}{\kappa}\left(I^{\text{horiz}} + I^{\text{fitting}}\right) \quad \text{for the system of Fig.3-2(b), or} \qquad (3.12)$$

$$\left(T^{\text{vert}}\right)^2 = \frac{4\pi^2}{\kappa}\left(I^{\text{vert}} + I^{\text{fitting}}\right) \quad \text{for the system of Fig.3-2(c).} \qquad (3.13)$$

Again, we use a formula of the difference

$$\left(T^{\text{horiz}}\right)^2 - \left(T^{\text{vert}}\right)^2 = \frac{4\pi^2}{\kappa}\left(I^{\text{horiz}} - I^{\text{vert}}\right) = \frac{\pi^2 M}{3\kappa}\left(3a^2 + 3b^2 - c^2\right). \qquad (3.14)$$

The last right side expression is derived using eq.3.09.

Then, without evaluating I^{fitting}, the modulus of rigidity G can be obtained through the definition of $\kappa = \pi G R^4/2L$ in eq.3.06 as

$$G = \frac{2\pi ML}{3R^4}\frac{3a^2+3b^2-c^2}{(T^{\text{horiz}})^2-(T^{\text{vert}})^2}. \tag{3.15}$$

Apparatus and Procedures:

1. In order to evaluate the periods accurately, a considerable number of oscillations should be sustained. Then, only the motion with small rotational amplitudes can be subject to the measurements. Even the rotational displacement (angle) of the object is quite small, a mirror which is mounted on the object reflects a ray of light which reaches a very wide range of places. This makes possible to read the angle precisely. This is a technique so-called "optical lever".

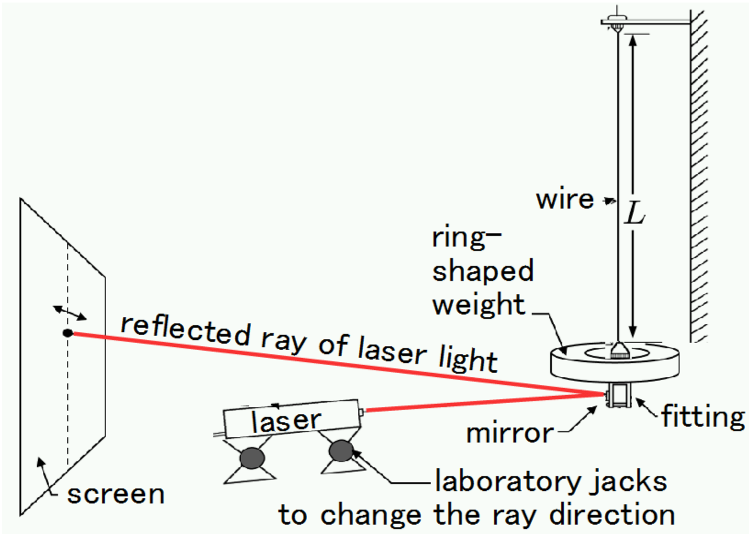


Fig. 3–3. Overall apparatus of the torsional pendulumn.

In this experiment, a laser beam is used to put an illuminated point on a screen. The mirror is mounted at a side of the fitting for the weight (Fig.3-2(a)). Overall apparatus is shown in Fig.3-3.

2. Before placing the weight, the orientation of the fitting on which the mirror is mounted should be adjusted by refastening the chuck (E in Fig.3-2(a)) in order to reflect the laser light toward the point just above the laser.

3. Place the weight horizontally (Fig.3-2(b)), with high adjustment of its position on the fitting in order to meet the center of mass of the weight and the axis of the wire. This procedure removes the additional modes of oscillation disturbing this experiment.

4. Switch on the laser and find the spot of the laser on the screen, the place of which should be adjusted by the laboratory jacks (upward or downward) and by the laser itself (to the right or left).

5. Start the oscillation with a small initial rotational angle, for example, 90 degree. Confirm that the spot oscillates horizontally. Shift the screen so that its central line is roughly coincide with the center of the oscillation.

6. Measure $10T^{\text{horiz}}$ by reading of the split times of 10 oscillations for 10 times without any break, so the consecutive 100 oscillations are investigated. Here, one oscillation must be counted for the back-and-forth motion. Calculate the average of $10T^{\text{horiz}}$ using the following idea.

If one have the data $x_0, x_1, x_2, \cdots, x_{10}$ that are consecutive 11 readings, ten substitutions $x_1 - x_0,\ x_2 - x_1,\ x_3 - x_2, \cdots, x_{10} - x_9$ should not be simply averaged, because

$$\frac{1}{10}\{(x_1 - x_0) + (x_2 - x_1) + (x_3 - x_2) + \cdots + (x_{10} - x_9)\} = \frac{1}{10}(x_{10} - x_0)$$

means that the remaining nine data are meaningless. Instead, an improved average

$$\bar{x} = \frac{1}{6}\left\{\frac{x_5 - x_0}{5 - 0} + \frac{x_6 - x_1}{6 - 1} + \cdots + \frac{x_{10} - x_5}{10 - 5}\right\}$$

is highly recommended.

7. Place the weight as shown in Fig.3.2(c) and measure $10T^{\rm vert}$ by the similar procedures.

8. Measure the length of the wire L for three times with a tape measure. Measure the diameter of the wire $2R$ for six times (at three points along the wire and two orthogonal directions for every points) using a micrometer.

9. Measure the inner diameter $2a$ and the outer diameter $2b$ of the weight for four times along the various directions, and measure the thickness c for four times at various places on the weight.

10. Calculate the modulus of rigidity G with eq.3.15 using the appropriate values of averages, i.e., $\overline{T^{\rm horiz}}$, $\overline{T^{\rm vert}}$, $\overline{L}$, $\overline{R}$, $\overline{a}$, $\overline{b}$, $\overline{c}$, and mass of the weight M which is indicated on the surface.

Sample Data:

Number of oscillations	Split time [s]	Number of oscillations	Split time [s]	Substitution $50T^{\rm horiz}$ [s]
0	000.00	50	248.04	248.04
10	049.52	60	297.56	248.04
20	099.14	70	347.10	247.96
30	148.85	80	396.81	247.96
40	198.39	90	446.40	248.01
50	248.04	100	495.99	247.95
			average of $50T^{\rm horiz}$	247.99 s
			$\overline{T^{\rm horiz}}$	4.960 s

$\overline{T^{\rm horiz}}$	$\overline{T^{\rm vert}}$	$\overline{L}$	$\overline{R}$
4.960 s	3.622 s	1.125 m	7.605×10^{-4} m
$\overline{a}$	$\overline{b}$	$\overline{c}$	M
6.503×10^{-2} m	9.517×10^{-2} m	3.035×10^{-2} m	3.106 kg

$G = 7.416 \times 10^{10}\ {\rm N/m^2}$

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