Electron

Except for a vacuum, huge number of atoms exist in any space. Occasionally some atoms loose one or two or more electrons by atom-atom collisions, which occur quite frequently when things are rubbed each other. Then they behave as ions or charged particles. A remarkable characteristic of the charged particle appears in the motion under electromagnetic fields. **The line of trajectories of the projected charged particles are curved due to the the electric or magnetic forces**. The examples of application includes an ink-jet printer in which the electrostatic force enables to adjust the location of the droplets reaching on a paper, and a cathode ray tube (CRT, or Braun tube) in which an electron beam (also called **cathode ray**) irradiates a spot on a screen and the place of the spot is controlled by use of the electric and magnetic fields.

Histrically, just after the high vacuum was achieved, electric conduction through the vacuum in the form of the cathode ray was found. The ray went though a thin sheet of metal through which any atoms can not penetrate, and seemed to rotate a thicker-metal wheel by the collision. Around the end of 19th century, scientists had a discussion about what really the ray that brings the kinetic energy and moment is. Indeed there was a theory of radiation, or a kind of a wave, like sound or light.

However, many scientists believed that the ray consisted of tiny fragents of matter which should have mass. The reason was that there were a lot of evidences that showed that the components of the cathode ray was charged negatively. The electric and magnetic fields deflected the ray in the way like that. The metal which was irradiated by the ray charged negatively. The ray flowed out only from the metal that was connected to the negative electrode, i.e., the cathode, of the high-voltage source.

Then **the specific charge, a chage to mass quotient, of the cathode ray** was investigated. As a result, the trajectory of the ray indicates that any fragments of the ray have a definite ratio of the charge to the mass. Additionally, the Coulomb repulsion force among any parts of fragments must be so strong that the fragments should be broken up completely. An **electron** was coined for a piece of it.

The charge of one single electron is now known to play a role as a unit for evaluating any microscopic amount of charges. The charge of an electron, e in symbol, is referred to as **elementary charge** and known to be $e = 1.602\ 176\ 620\ 8(98) \times 10^{-19}$ Coulombs. One should not think it is too small. A number, $6.022\ 140\ 857 \times 10^{23}$ (the Avogadro's number) of particles are known to correspond to a macroscopic amount of matters. Then, one mole of electrons has totally 96 485.332 88 Coulombs (so-called Faraday constant), that is roughly the same as a 1-Ampere electric current which runs for 1 day and 3 hours. The mass of the electron is known to be unimaginably small; $m = 9.109\ 383\ 56 \times 10^{-31}$ kg, about one-half of one-thousandth of the mass of hydrogen atom (the lightest atom in the periodic table). It was only obtainable through the" measurements of the specific charge e/m and the elementary charge e.

Some of the histrical storyteller mentioned that Sir Joseph John Thomson *discovered* the electron, but it is a physics model which was established by many researchers hard works. He is a Nobel laureate of 1906 for the contribution to the experimental investigation of the conduction of electricity of gases. His son, Sir George Paget Thomson is also a a Nobel laureate of 1937 for the contribution to the experimental discovery of the diffraction of electrons by crystals. Then, the electron beam with the momentum mv is now known to behave as a wave with wavelength $\lambda = h/mv$, where $h = 6.626\ 070\ 040 \times 10^{-34}$ J s is Planck constant.

Experiment 1: Specific Charge of Electron e/m(the Electron Charge to Mass Quotient)

Purpose: To understand the mechanism of circular motion of a charged particle in magnetic fields. To evaluate the specific charge of electron from the measurements of radius of the circular orbit.

Theory 1-1: Accelerator of Electron

In the vacuum space, electrons are able to emitted from the heated cathode metal. A voltage $+V$ [V] is applied on a plate which faces to the cathode at 0 V. The electrostatic field with a uniform intensity $E = V/d$ [V/m] is spread over the space enclosed in the plate and the cathode. The electron with a charge $q = -e$ [C] and mass m [kg] is accerelated to obtain the speed v [m/s] until reaching the plate, where the gain of kinetic energy is $\frac{1}{2}mv^2 = eV$, therefore

$$v = \sqrt{\frac{2eV}{m}}. \tag{1-01}$$

The plate has a hole or a gap from which the accelerated electrons are projected to the outer space. If the applied voltage is, for example, $V = 300$ V, the velosity of the accelerated electron becomes

$$v = \sqrt{\frac{2 \times 1.60 \times 10^{-19}\,\mathrm{C} \times 300\,\mathrm{V}}{9.11 \times 10^{-31}\,\mathrm{kg}}} \sim 1 \times 10^7\,\mathrm{m/s}. \tag{1-02}$$

As reference, the corresponding wavelength is $\lambda = h/mv = h/\sqrt{2meV} \sim 0.07\,\mathrm{nm}$. Diffraction of a wave with such a short wavelength is not easy to be observed.

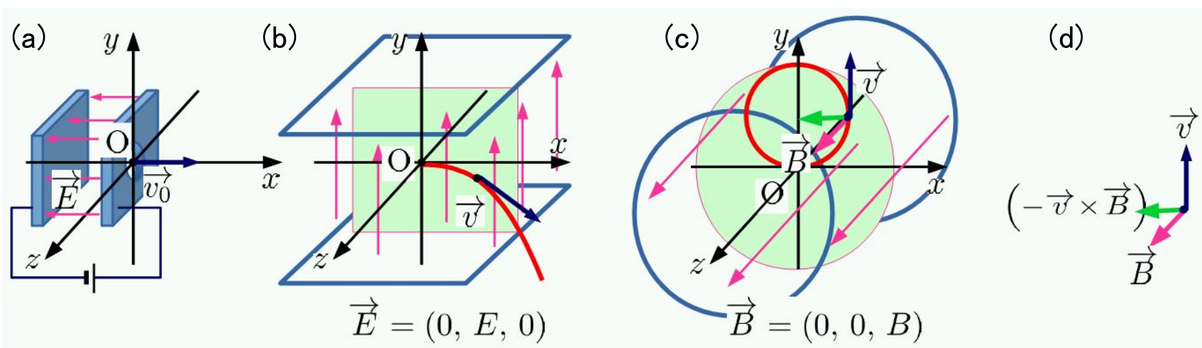

Fig 1 (a) The accelerator of the electron, (b) the motion in the electric field directed along the y axis, and (c) that in the magnetic field along the z axis. (d) The vectors $\vec{v}$, $\vec{B}$, and $(-\vec{v} \times \vec{B})$.

Theory 1-2: Lorentz force

Let us think about a vacuum space with a uniform electric field $\vec{E} = (0,\ E,\ 0)$ [V/m] or a uniform magnetic field $\vec{B} = (0,\ 0,\ B)$ [T] and a particle with charge $q = -e$ [C] runs with the initial velocity vector $\vec{v_0} = (v_{0x},\ v_{0y},\ v_{0z})$.

When $B = 0$, the electric force is given as

$$\vec{f_{\rm E}} = (0,\ -eE,\ 0) \tag{1-03}$$

then the acceleration vector is given as a constant $\vec{a} = \vec{f}/m = (0,\ -eE/m,\ 0)$, so that the motion is the horizontal projection and its trajectory becomes a curve of the parabolla.

In the case $E = 0$, the magnetic force (especially it is called as **Lorentz force**) is known to be written with the velocity $\vec{v} = (v_x,\ v_y,\ v_z)$ and magnetic field $\vec{B} = (0,\ 0,\ B)$ as

$$\vec{f_{\rm B}} = -e\vec{v} \times \vec{B} = -e(v_y B,\ -v_x B,\ 0). \tag{1-04}$$

Then each component of the accerelation vector $\vec{a} = (a_x,\ a_y,\ a_z) = \left(\frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt}\right)$ is

$$\frac{dv_x}{dt} = -\frac{e}{m} v_y B, \tag{1-05a}$$

$$\frac{dv_y}{dt} = \frac{e}{m} v_x B, \tag{1-05b}$$

$$\frac{dv_z}{dt} = 0. \tag{1-05c}$$

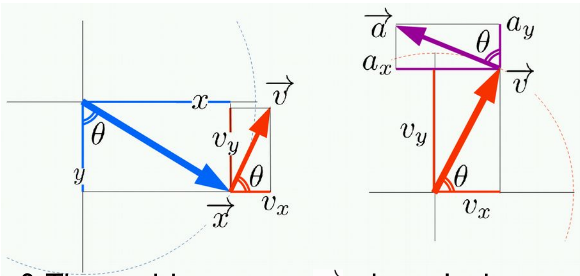

Fig 2 The position vector $\vec{x}$, the velocity vector $\vec{v}$, and the acceleration vector $\vec{a}$ in a circular motion.

The velocity component v_z along z axis is apparantly constant. The other component along the xy plane, $v_{xy} = \sqrt{{v_x}^2 + {v_y}^2}$, is also constant because it satisfies that

$$\frac{dv_{xy}}{dt} = \frac{d({v_x}^2 + {v_y}^2)^{1/2}}{dt} = \frac{1}{2v_{xy}} \frac{d(v_x^2 + v_y^2)}{dt} = \frac{1}{v_{xy}} \left(v_x \frac{dv_x}{dt} + v_y \frac{dv_y}{dt} \right) = 0. \tag{1-06}$$

Here the relation of eq.1-05a and eq.1-05b are used. Similary, as for $v = \sqrt{{v_{xy}}^2 + {v_z}^2}$,

$$\frac{dv}{dt} = \frac{d({v_{xy}}^2 + {v_z}^2)^{1/2}}{dt} = \frac{1}{2v} \frac{d({v_{xy}}^2 + {v_z}^2)}{dt} = \frac{1}{v} \left(v_{xy} \frac{dv_{xy}}{dt} + v_z \frac{dv_z}{dt} \right) = 0. \tag{1-07}$$

Then, the motion should be a uniform screw with a uniform circular motion in the xy plane coupled with a constant velocity motion perpendicular to the xy plane. If the initial velocity $\vec{v_0} = (v_{0x},\ v_{0y},\ v_{0z})$ has completely perpendicular to the magnetic field, i.e., $v_{0z} = 0$, the trajectory of motion is closed in a circle just on the xy plane.

In the uniform circular motion, the radius of the trajectory r is given in a formula of magnitude of the centripetal acceleration a_{xy} with the velocity squared ${v_{xy}}^2 = {v_x}^2 + {v_y}^2$, that is

$$a_{xy} = \frac{{v_{xy}}^2}{r}. \tag{1-07}$$

Because the acceleration of the formula in eqs.1-05 gives

$$a_{xy} = \sqrt{\left(\frac{dv_x}{dt}\right)^2 + \left(\frac{dv_y}{dt}\right)^2} = \frac{e}{m} v_{xy} B, \tag{1-08}$$

a relation of the specific charge $\frac{e}{m}$ and the radius r is given using eq.1-07 and eq.1-08

$$\frac{e}{m} = \frac{v_{xy}}{rB} \tag{1-09}$$

The magnitude of the velocity v is constant so that it should be given in the accelerator with the voltage V. When $\vec{v} \perp \vec{B}$, $v_{xy} = v$ which is given in eq.1-01, therefore

$$\frac{e}{m} = \frac{1}{rB}\sqrt{2V\frac{e}{m}}, \tag{1-10}$$

accordingly,

$$\frac{e}{m} = \frac{2V}{r^2B^2}. \tag{1-11}$$

Theory 1-3: Fitting Data Using the Least-Mean-Squares (LMS) Method

The eq.1-11 is considered that the measured value $y = r^2$ is a linear function of the condition $x = V$ with an unknown constant $a = 2m/eB^2$. We rewrite the equation as

$$y = ax + b. \tag{1-12}$$

Here the unknown constant b is included in eq.1-12 because it sometimes happens that an extrapolated value of $y = 0$ is expected under a finite value of $x = -b/a (\neq 0)$. The equation eq.1-12 expresses a physics model that should be debatable. Adding the parameter b is just an example of accepting a modification of the model.

One simple way to determine the parameters (a and b) in the model (eq.1-12) is to make a graph and to plot all (N pieces) of the data $(x_1, y_1), (x_2, y_2), \cdots, (x_i, y_i), \cdots, (x_N, y_N)$ on it, and to draw a line which represent the model. Another way with a reliable procedure is so-called the Least-Mean-Squares (LMS) method. It is designed to determine the best values of a and b which minimize the sum of the square of residual Δy_i, which is defined as the difference between measured value of y_i and its prediction $ax_i + b$. The sum S is given as

$$S = \sum_{i=1}^{N} (y_i - ax_i - b)^2, \tag{1-13}$$

then a and b necessarily meet the following conditions

$$\frac{\partial S}{\partial a} = \sum_{i=1}^{N} -2x_i(y_i - ax_i - b) = 0 \quad \text{or,} \left(\sum_{i=1}^{N} x_i^2\right) a + \left(\sum_{i=1}^{N} x_i\right) b = \left(\sum_{i=1}^{N} x_i y_i\right) \tag{1-14}$$

$$\frac{\partial S}{\partial b} = \sum_{i=1}^{N} -2(y_i - ax_i - b) = 0 \quad \text{or,} \left(\sum_{i=1}^{N} x_i\right) a + \left(\sum_{i=1}^{N} 1\right) b = \left(\sum_{i=1}^{N} y_i\right) \tag{1-15}$$

Then, what you need is to calculate each coefficient in equations eq.1-14 and eq.1-15 and just solve them. If

$$D = \left(\sum_{i=1}^{N} x_i^2\right)\left(\sum_{i=1}^{N} 1\right) - \left(\sum_{i=1}^{N} x_i\right)^2 \tag{1-16}$$

is not zero, then the solution of the system of equation is obtained as

$$a = \frac{1}{D}\left\{\left(\sum_{i=1}^{N} x_i y_i\right)\left(\sum_{i=1}^{N} 1\right) - \left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} y_i\right)\right\}, \quad (1\text{-}17)$$

$$b = \frac{1}{D}\left\{\left(\sum_{i=1}^{N} x_i^2\right)\left(\sum_{i=1}^{N} y_i\right) - \left(\sum_{i=1}^{N} x_i y_i\right)\left(\sum_{i=1}^{N} x_i\right)\right\}. \quad (1\text{-}18)$$

Theory 1-4: Helmhortz Coil

Two circular flows of parallel currents with a common axis make a rather uniform magnetic field in the middle of circles. The uniformity is maximized when the distance between two circles is arranged to be just the same as their radius. The apparatus with such two circular currents is called Helmhortz coil, which is widely used because its center is easy to access in order to put samples or sensors. A realistic Helmhortz coil with a circular-shaped section of currents with its diameter D and the coil radius R is shown in Fig.3.

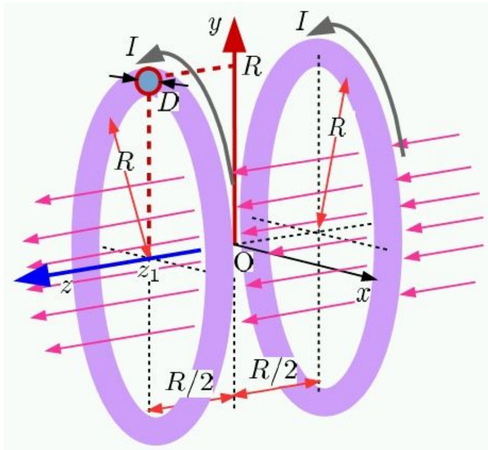

Fig 3 A Helmhortz coil with the coil radius R and the circular-shaped current with its diameter D.

If $D = 0$, the magnetic field on the z axis is easily derived from Biot-Savart law of magnetic fields owing to the current elements. Here we discuss only the magnetic fields on the central plane at $z = 0$. Theoretically,

$$B_z(r = 0) = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 I}{R} \quad (1\text{-}19)$$

where the pearmiability(magnetic constant) is $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$. When I =10 A, $R = 18$ cm, for example, the formula eq.1-19 gives $B_z(0) = 4.995 \times 10^{-5}\,\mathrm{T}$ ($D = 0$).

For $D \neq 0$, only numerical calculations are available. For $D = 2\,\mathrm{cm}$ and $I = 10$ A, $R =$ 18 cm under bounday conditions of $B \parallel z$ at $z = 0$ and an axial symmetry around $r = 0$, a solution of a finite elementary method (FEM) showed that $B_z(0) = 4.84 \times 10^{-5}\,\mathrm{T}$ and considerable uniformity around $(z, r) = (0, 0)$, as shown in Fig.4.

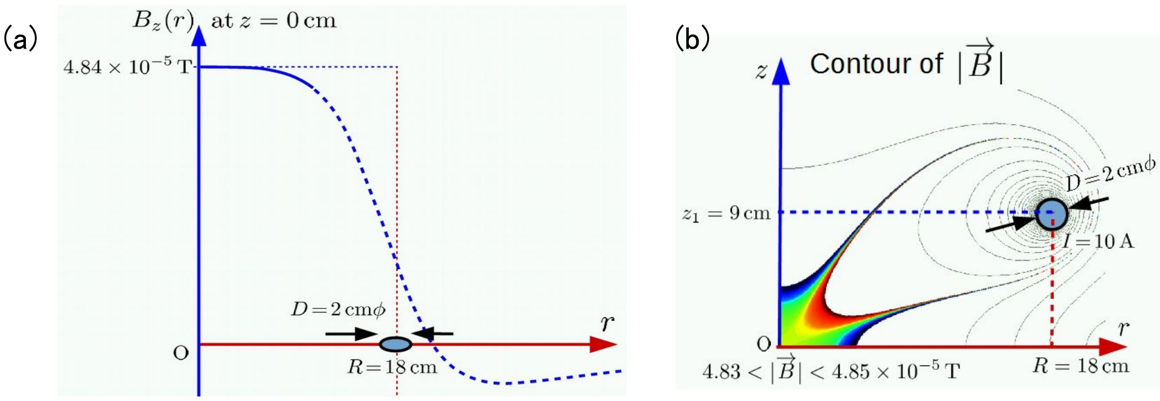

Fig 4 A distribution of the magneic flux density B on the central plane (z = 0) of Helmhortz coil from a solution of FEM. (a) The r dependence of the z component of $Bz(r)$, (b) a contour map of magnitude of $\vec{B}$.

Apparatus and Procedures:

1. Check knobs A on the current source and B on the accelerator, both of which should be minimized by turning all the way in the counterclockwise direction. Switch on the power of the whole system. Within a few seconds, a heater of the accerelater in the vacuum tube is heated at a red heat.

2. Turn the knob B slowly to increase the acceleration voltage. A bright line of the electron beam appears in the vacuum tube.

3. Swith on the current source and turn the knob A slowly until an angle of 3 o'clock. Check increase on the ammeter and voltmeter installed on the current source.

4. Turn the knob C in order to increase the output curret I for the coil. The magnetic field B around $(z, r) = (0, 0)$ is given by use of the curent I with a formula

$$B = \left(7.8 \times 10^{-4}\,[\mathrm{T/A}]\right) I \tag{1-20}$$

which is given by the manufacturer. A red indicator lamp shows that the mode is changed to a Constant Current (CC) mode. The shape of the trajectory of the electron beam changes to be curved. If the shape becomes screw-like, you need to change the angle of the accelerator to the magnetic field by slightly rotating the vacuum tube.

5. Measure the radius r of the trajectory circle under some acceleration voltage V. Changing the voltage V but keeping the current I constant, take five data of (r, V). Take care of the rated values of the apparatus ($I_{\max} = 12\,\mathrm{A}$, $V_{\max} = 300\,\mathrm{V}$). If the trajectory becomes a thick line, it is recommended to read the brightest place of in the line. Because the scale is far from the trajectory, the line of sight for reading should be perfectly perpendicular to the scale.

6. When the measurements are finished, turn the knobs all the way in the counterclockwise direction in order to minimize I and V.

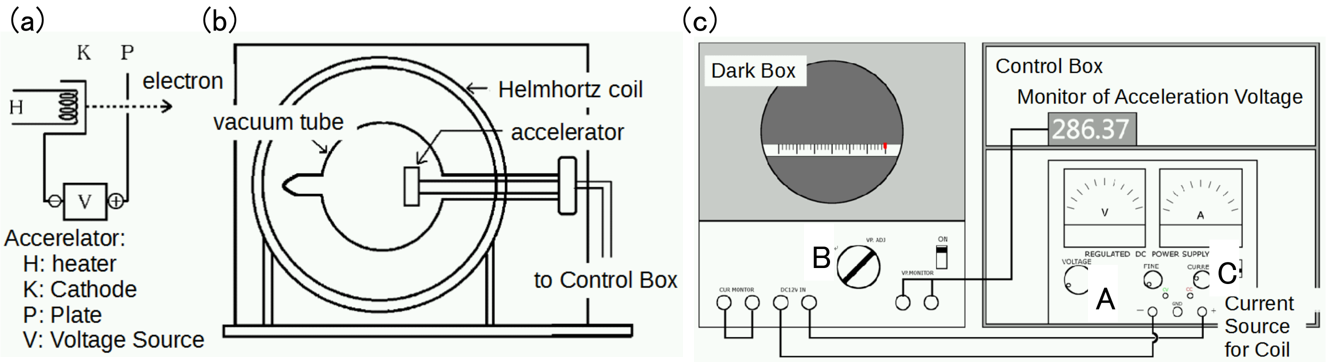

Fig 5 (a) Accelerator of the electron, (b) Dark box, (c) Dark box and Control Box

Version 01 October 28, 2015
by Tetsuya Kato, Prof. Dr., Faculty of Education, Chiba University
E-mail: tkato@faculty.chiba-u.jp